

1. F. G. TRICOMI, *Lezioni sulle funzioni ipergeometriche confluenti*, Gheroni, Torino, 1952.
2. H. BUCHHOLZ, *Die konfluente hypergeometrische Funktion mit besonderer Berücksichtigung ihrer Anwendungen*, Springer, Berlin, 1953.
3. A. ERDÉLYI, W. MAGNUS, F. OBERHETTINGER, & F. G. TRICOMI, *Higher Transcendental Functions*, v. 1. Chapter VI, McGraw-Hill, New York, 1953.
4. RMT 46, *MTAC*, v. 12, 1958, pp. 86–88.

23[M, X].—G. DOETSCH, *Einführung in Theorie und Anwendung der Laplace-Transformation*, Birkhäuser Verlag, Basel, Switzerland, 1958, 301 p., 24 cm. Price SFr 39.40.

This book forms an excellent introduction to the subject of Laplace transformations in one dimension, written by one of the leading experts in the field. From his wide knowledge of both the theoretical and applied aspects of the subject, the author has written a very readable, rigorous exposition which also indicates relations with appropriate physical concepts.

After a general introduction, the basic properties of the Laplace transform are developed in ten short chapters. Included are discussions of half-planes of convergence, uniqueness of the inverse, analytic properties of the transform, the effect of a linear transformation of the independent variable on the transform, the effect of differentiation and integration, and the transformation of a convolution. Because each chapter is devoted to a separate topic, the book is very useful for reference purposes. Many examples of specific transforms are given.

The next four chapters deal with the application of Laplace transforms to the following problems: the initial-value problem in ordinary differential equations with constant coefficients; the solution of differential equations for special input functions; homogeneous and non-homogeneous systems of differential equations; the initial-value problem for difference equations.

The next group of chapters deals with further properties of the transform, some of which may not be familiar to the reader: the behavior of the transform at infinity; inversion formulas expressed as integrals along vertical lines, as integrals along deformed paths in the complex plane, and as series of residues; conditions for the representation of a function as a transform; functions given as the sums of series of transforms; the analogue of Parseval's formula and transforms of products; and the asymptotic behavior of the transform and of the "original" function.

The book concludes with three chapters on further applications of the Laplace transform to differential equations with variable coefficients, to simple partial differential equations, and to certain integral equations.

A useful feature of the book is the inclusion of necessary background material, particularly in the later chapters.

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24[S, X].—G. I. MARCHUK, *Numerical Methods for Nuclear Reactor Calculations*, (An English translation of a work originally published in Russian as Supplement Nos. 3–4 of the Soviet Journal of Atomic Energy, *Atomnaya Énergiya*. Atomic Press, Moscow, 1958.) Consultants Bureau, Inc., New York, 1959, 293 p. 28 cm. Price \$60.00.

A more accurate title for this book would have been *Numerical Methods for*

Nuclear Reactor Physics Calculations. As stated by the author, "the book is an attempt at a more or less systematic exposition of numerical methods for the calculation of thermal, intermediate, and fast neutron reactors. Particular attention is devoted to the problems of critical mass, the space-energy neutron flux distribution, and the neutron importance." This is the first book to become available in English that is entirely devoted to this area of calculation, and as such is a helpful addition to our literature.

The book is clearly not addressed to mathematicians. A familiarity with nuclear reactor theory is assumed; terms such as "cross section" are used without definition. The mathematician will probably be disturbed by such statements as "the integrands in the resulting two terms are now continuous, and they can therefore be expanded in a Taylor's series about r_k " (p. 106), or, "primarily, this is due to the fact that a second-order difference equation has two eigenvalues, one of which is positive" (p. 122). Emphasis throughout the book is on the derivation of equations and the mechanics of their solution. The stability of a number of the numerical methods against the exponential growth of round-off errors is considered. Problems related to the existence of solutions and the accuracy with which solutions of the various approximate equations represent the desired solutions are mentioned only occasionally.

In the foreword the author mentions that the numerical methods were "first tested on a large quantity of theoretical and experimental data and were later used in operation." It is unfortunate that the results of some of these calculations were not included in the book. Results of specific calculations are not given. For example, in Chapter VIII, various sets of finite-difference equations are set up to approximate the diffusion equation. Some of these difference equations are more accurate than others. However, no discussion of the actual errors incurred when these various approximations are used is presented other than a statement, "with a sufficiently fine network good accuracy can be obtained." This would have been an excellent place to include numerical results showing the kind of accuracy that could be attained when the various approximate equations are used in typical specific problems.

A useful feature of this book is that corresponding to every set of equations derived, an adjoint set of equations is also obtained.

In the first chapter a basic set of transport equations is derived: a "slowing-down" equation in which the assumption is made that the moderator nuclei are at rest while the neutrons are slowed down by elastic collisions, and a "thermal" equation in which the velocities of the moderator nuclei are assumed to have a Maxwellian distribution. In succeeding chapters, various sets of equations approximating this basic set are obtained. In the second chapter the diffusion approximation is derived following the method of R. E. Marshak, H. Brooks, and H. Hurwitz, Jr., published in *Nucleonics*, v. 4, 5 (1949), in which the angular dependence of the neutron flux and the collision function are both approximated by retaining the first two terms in a series expansion involving spherical functions. The corresponding boundary conditions are also derived. In the third chapter the diffusion-age approximation is obtained when the energy dependence of the flux is approximated by retaining the first few terms of a Taylor series expansion. Here again, the corresponding boundary conditions are derived. In the fourth chapter further approximations to the basic transport equation are derived which take more accurate account of energy dependence than the diffusion-age approximation,

and are therefore applicable in cases where there is stronger absorption of slowing-down neutrons. Equations are derived in which resonance absorption is accounted for based on a method suggested by Wigner. Also, the Greuling-Goertzel equations for the slowing-down of neutrons in an inhomogeneous medium are obtained.

In Chapter V exact solutions are given of the basic and adjoint equations of the diffusion-age approximation for a homogeneous, unreflected reactor with an extrapolated boundary which is energy-independent. Also, for this special case the convergence of "the method of successive approximations" for finding the criticality factor is established. "The method of successive approximations" means here that at each step, assuming an estimate of the fission source, the corresponding neutron flux is calculated, and then using this set of flux values, a new estimate of the fission source is obtained. The criticality factor is obtained using ratios of successive source estimates.

In Chapter VI various sets of multigroup equations are derived which are applicable primarily to thermal reactors where weak absorption of slowing-down neutrons is assumed; in Chapter VII various sets of multigroup equations are derived which are applicable primarily to intermediate reactors where strong absorption of slowing-down neutrons must be considered.

Chapter VIII contains the derivation of various sets of three-point difference equations to approximate the one-dimensional diffusion equation in plane, cylindrical, and spherical geometries. In addition, five-point difference equations are obtained for the two-dimensional diffusion equation in plane (x, y) and cylindrical (r, z) geometries.

In Chapter IX various methods are discussed for solving sets of finite-difference diffusion equations of the types set up in the previous chapter. To solve the three-point difference equation a well-known factorization technique is described, and its stability against the growth of round-off errors is considered. For the solution of the five-point difference equations an analogous factorization technique, as well as a simple iteration procedure, and a third method suggested by N. I. Buleev are considered. Error criteria which can be used to estimate the accuracy of results are also mentioned.

Chapter X contains a discussion of the "net" method for obtaining approximate solutions of the differential slowing-down equation in one spatial dimension. Difference equations are set up to approximate the differential equation and solved for the unknown function in one energy interval in terms of values computed for the preceding interval. Both explicit and implicit schemes are discussed. Their stability is investigated by applying the von Neumann stability criterion.

In Chapter XI perturbation theory is applied to obtain approximate formulas giving the change in the criticality factor when small changes are made in values of the various physical parameters. This is done starting with the basic equations of Chapter I as well as with the various equations obtained in later chapters to approximate these basic equations. Some further applications of perturbation theory are discussed such as the calculation of the reactivity equivalent of a control rod in the center of the core of a cylindrical reactor, as well as for a whole system of control rods in such a reactor.

Chapter XII on heterogeneous effects in nuclear reactors contains descriptions of the Gurevich-Pomeranchuk theory of resonance absorption applicable in calculating the effective resonance integral for "thin" U^{238} blocks, the Wigner theory for

calculating the resonance integral for "thick" blocks, and Orlov's extension of Wigner's method giving a formula for the effective resonance integral which can be used for U^{238} blocks of any thickness. Further, a method is given which was developed by Marchuk and Orlov for calculating the resonance capture of neutrons in a plane lattice of uranium blocks. Sedel'nikov's extension of the Gurevich-Pomeranchuk theory to take account of self-shielding in intermediate neutron reactors is described. Effective boundary conditions for "black" and "grey" bodies are considered. Finally, the Wigner-Seitz method is applied to a cell of a heterogeneous reactor to obtain effective constants for an equivalent homogeneous reactor.

Chapter XIII contains a discussion of the calculation of the neutron flux in fast nuclear reactors. The transport equation including terms for inelastic scattering is given. Corresponding multigroup equations are obtained. For solving these equations both the method of spherical harmonics and Carlson's S_n method are described.

A final chapter on calculations for intermediate and thermal reactors with hydrogenous moderators is included. This chapter was added after the original manuscript had been prepared for publication.

It is unfortunate that this book was not more carefully edited. For example, there is confusion in the use of the terms 'flux' and 'current'; on page 104 the following statement appears, "we wish to find a solution of (30.1) which has a continuous flux

$$I = r^a D \frac{d\phi}{dr} ."$$

Throughout the text, the equations which are usually referred to as "transport" equations are called "kinetic" equations. Too frequently proper names are spelled phonetically as a result of the transliteration process, rather than with their usual English spelling; for example, R. E. Marshak appears as R. E. Marchak (p. 6), R. Ehrlich appears as R. Erlich (p. 6), Neumann appears as Neiman (p. 142), etc. Also, it is unfortunate that the publishers did not take more care in their representation of symbols. The equations were apparently reproduced photographically. A parameter represented by a script letter in an equation is frequently represented in another form in the text. On pages 132 and 134 a parameter which appears in the equations as a "chi" is represented as a "kappa" in the text.

This book is a very useful addition to our literature. It is hoped that its presence and imperfections will act as a stimulus for the publication of another book in the area of numerical methods for nuclear reactor calculations which will give a more satisfying mathematical treatment of this subject, and which will be made available at a more reasonable price.

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25[W].—E. BURGER, *Einführung in die Theorie der Spiele*, Walter de Gruyter & Co., Berlin, 1959, 169 p., 23 cm. Price DM 28.

This is the first book on the theory of games to appear in the German language. The author, professor of mathematics at the University of Frankfurt, has previously